

Theoretical study of waves diffracted from asymmetrical profile lamellar gratings

Sonali Chakrabarti^{1,2}, Sutapa Chatterjee¹, Utpal Chatteropadhyay¹ and J N Chakravorty^{1*}

¹Centre for Space Physics, 43, Chalanika, Garia Station Main Road, Kolkata-700 084, India

²Maharaja Manindra Chandra College, 2, Ramkanto Bose Street, Kolkata-700 003, India

E-mail : space_phys@vsnl.com

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Abstract : Diffraction from a microwave asymmetrical lamellar grating (ALG) has been thoroughly investigated. Expressions for intensity, dispersive and resolving power for such a grating have been derived. It is anticipated that such gratings with asymmetrical structure may be used with significant advantage for taking observations in both optical and quasi-optical region. By a suitable choice of geometrical parameters, its applicability may as well be extended to the millimeter and microwave bands which will have significant applications in instruments for astronomical observations and microwave space-communication.

Keywords : Microwave diffraction, millimeter waves, lamellar gratings.

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1. Introduction

Research in millimeter (mm) waves and microwaves is being carried out in the forefront in radio astronomy, communication and various other fields for their significant operational advantage like better penetration properties in adverse weather conditions, over their higher frequency counterparts. However, a great set back for the present day mm and microwaves instruments is that they do not have the extremely high resolutions as compared to the higher frequency devices operating in the IR, visible or the UV region. The use of grating-based instruments can be an answer to the resolution problem for the mm and microwaves. In addition, they would have the ability of simultaneous recording of many spectral elements at a very low noise band-width. However, due to very low intensity and completely different penetration characteristics of these waves, gratings with special designs would be required. Indeed, in a series of papers [1-4], we have explored the characteristics of diffracted waves from transmission and reflection echelon, echelette and

symmetric lamellar gratings. This paper presents a theoretical study of the intensity pattern, resolution and the other dispersion characteristics of an asymmetric profile lamellar grating, a special type of phase grating, which we anticipate, would be a good candidate for a mm or microwave grating instrument.

It is well known that an optical lamellar grating is a useful device for studying the diffraction effects in a near infra-red region [5]. The performance of such dielectric gratings has been analysed by using suitable perturbation methods [6,7] and also by electromagnetic theory [8]. Several optical principles are being used in the construction of microwave antenna [9-14]. Millimeter waves possess directional property and are found suitable for communication link [15], and an asymmetric lamellar grating may work very well in the mm wave region. With suitable modification, it may also be used as a microwave antenna in space communication. Its construction is simple and may be fabricated in the following way. A highly polished metallic surface may be suitably shaped to have

*Corresponding Author

changes from 1 cm to 5 cm along the x -axis and the y -axis respectively (Figure 5).

We note that the minimum values of a and b sufficient for an optimum intensity response for the wavelength $\lambda = 5$ mm are, $a \sim 3$ cm and $b \sim 1.4$ cm. Knowledge of the

$b = 1$ cm, $N = 30$ is shown in Figure 6. The wavelength changes from 1 mm to 10 mm along x -axis and θ changes from 5° to 15° along the y -axis.

From eq. (4), it follows that J is maximum when

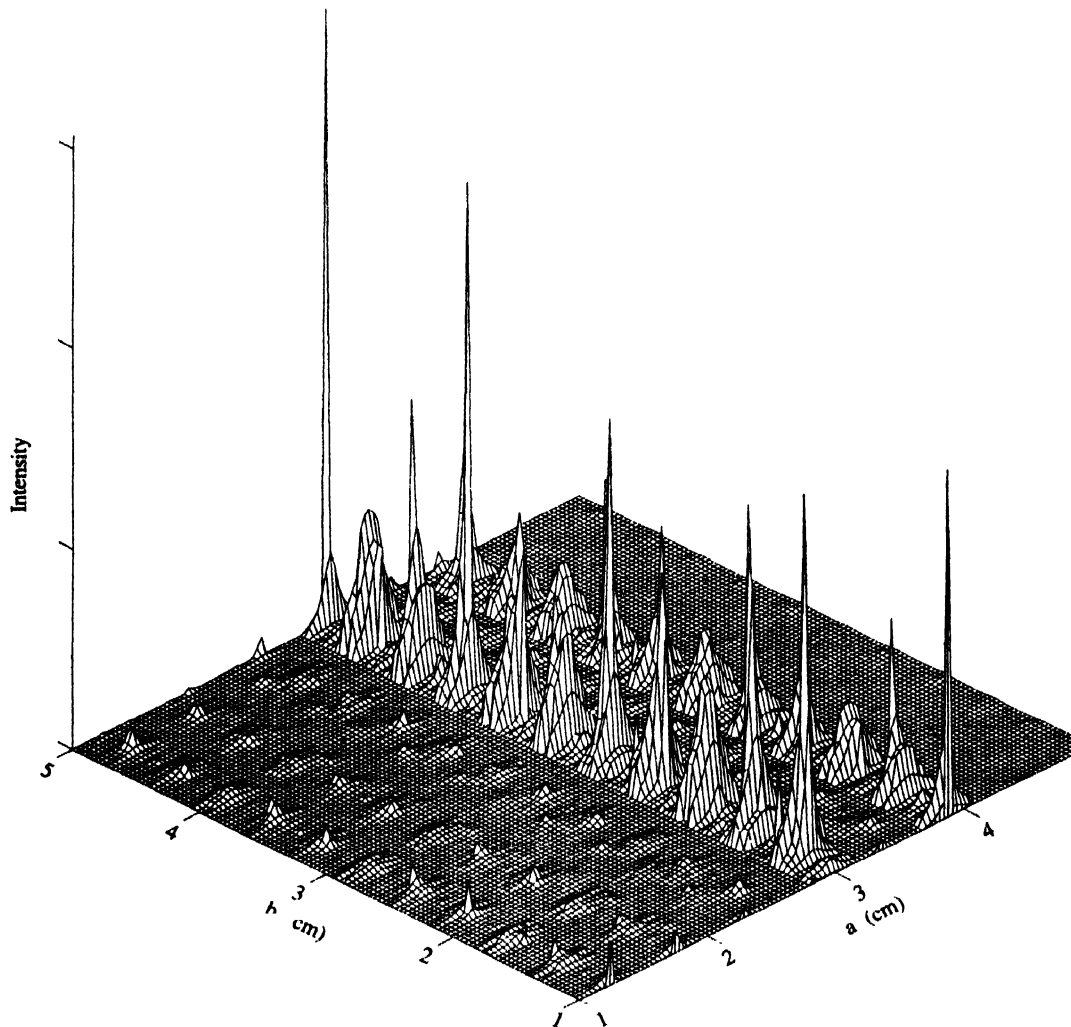


Figure 5. Intensity distribution (J) as a function of the geometrical parameters a and b of the grating, at $\lambda = 5$ mm. Note that J is more sensitive to the length a of each step than its height b .

minimum values of the parameters for the optimum intensity will reduce the construction cost and is important from the commercial point of view. Parameters for all operating wave lengths can be similarly worked out from the expression for J .

If the same grating needs to be used for different wavelengths, then from the 3 dimensional plot of J as a function of θ and λ with the geometrical parameters of the grating, the position of the detector to get optimum response for the required wavelength can be selected. A 3 dimensional plot of J (z -axis) with $a = 1.5$ cm,

$$\sin \theta = \frac{m\lambda}{3a}$$

$$\text{Therefore, } \cos \theta \, d\theta = \frac{m}{3a} \, d\lambda.$$

$$\text{or } \frac{d\theta}{d\lambda} = \frac{m}{3a \cos \theta}.$$

The above equation gives us the angular dispersion of an asymmetrical profile lamellar grating.

From the expression of J , the resolving power of an ALG can be calculated. The resolving power in this case, exhibits an interesting feature. It is observed that there is

From the expression for J , we have $J = 0$ when $\sin^2 3N\phi = 0$, that is

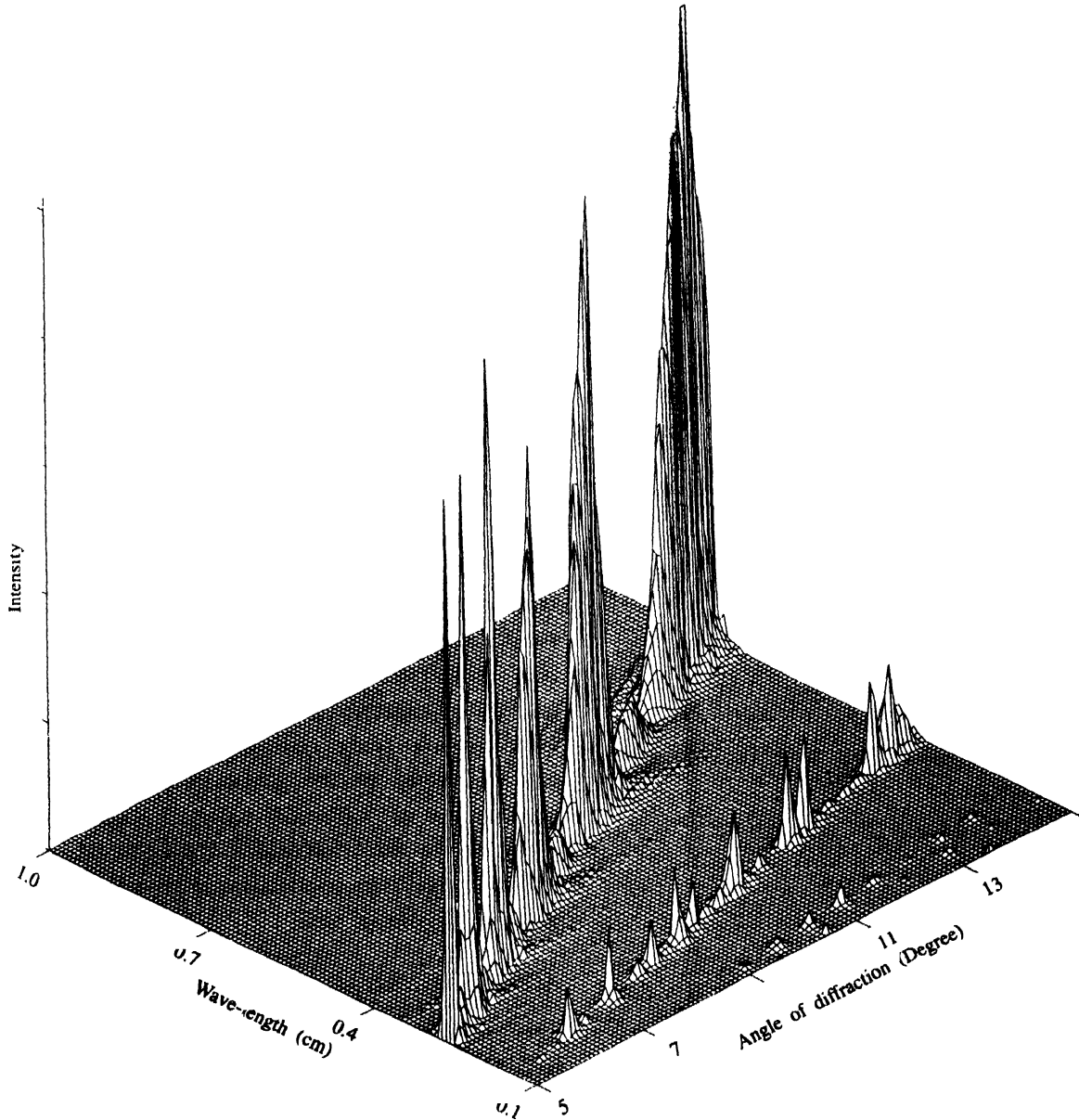


Figure 6. Intensity distribution (J) as a function of wavelength (λ) and diffraction angle (θ). Parameters are $a = 1.5$ cm, $b = 1$ cm and $N = 30$.

a critical value of N (say N_0), such that if $N > N_0$, the zero intensity nearest to the principal maximum is due to the term $\frac{\sin^2 3N\phi}{\sin^2 3\phi}$, and the resolving power is the usual mN . However if $N < N_0$, then zero intensity nearest to the principal maximum is due to the term $(2\cos \psi + 1)^2$ in the expression for J , and $\frac{\lambda}{d\lambda} \sim \frac{a \sin \theta}{2b}$. The calculation is quite simple and is shown below.

$$\sin \theta = \frac{m\lambda}{3Na} \quad (7)$$

where m is the order number of the minima. Also $J = 0$, when

$$(2\cos \psi + 1) = 0. \quad (8)$$

We will come back to this later.

J has a maximum value when $\sin^2 3\phi = 0$, that is

when

$$\sin \theta = \frac{m\lambda}{3a} \quad (9)$$

where m is the order number of the principal maxima.

Putting $\sin \theta = p$, the separation between a primary maximum of order m and a neighbouring minimum given by (7) is

$$\Delta p = \frac{m\lambda}{3Na} \quad (10)$$

If the wavelength is changed by $\Delta\lambda$, the m -th order principal maximum is displaced by

$$\Delta p' = \frac{m\Delta\lambda}{3a} \quad (11)$$

On the limit of resolution in the m -th order,

$$\frac{m\Delta\lambda}{3Na} = \frac{2b}{a}$$

$$\text{or } \frac{N}{\Delta\lambda} = mN. \quad (12)$$

Eq. (8) gives $J = 0$ when $(2\cos \psi + 1) = 0$, that is when $\psi = \frac{2\pi n}{3}$,

$$\text{or } \sin \theta = \frac{n\lambda}{3a} - \frac{2b}{a}$$

for small θ . Putting $\sin \theta = p$, the separation between a primary maximum and the next minimum caused by (7) is

$$\Delta p = \frac{2b}{a}. \quad (13)$$

Therefore, at the limit of resolution for this case,

$$\text{or } \frac{m\Delta\lambda}{3a} = \frac{2b}{a} \quad (14)$$

Since N is a comparatively large number, the resolving power given by (12) is definitely greater than that given

by (14). For a grating, the resolving power is governed by eq. (14) instead of (12) when Δp given by (13) is less than that given by (10). That is

$$\frac{2b}{a} < \frac{\lambda}{3Na} \quad (\text{for small } \theta)$$

$$\text{or } N < \frac{\lambda}{6b}.$$

Therefore, it follows that for an asymmetrical profile lamellar grating to have a high resolving power at a particular λ , the number of asymmetric profiles N must satisfy the relation

$$N > \frac{\lambda}{6b}$$

4. Concluding remarks

The above discussion gives a sufficient idea as to how to obtain optimum intensity and resolution with minimum values of grating parameters. Hence the construction cost of such instruments can be sensibly be reduced and make them economically viable. With proper choice of such ideal geometrical parameters, these asymmetric lamellar gratings are expected to be used with enormous advantage in microwave space communication, radar technology, remote sensing, missile guidance, radio astronomy and spectroscopy of molecular clouds in space. Thus we anticipate that these microwave grating based instruments will have significant applications in the field of astronomy and astrophysics as well.

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